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ABSTRACT

This document briefly reviews four areas of educational inquiry. The first section is concerned with definitions of the terms "spatial ability" and "visual imagery"; the second is concerned with training studies in which attempts have been made to improve spatial ability or to encourage greater use of visual imagery in problem solving; the third pertains to sex differences in spatial performance; and the fourth is concerned with studies investigating relationships between spatial ability, visual imagery and mathematical performance. It is concluded that not only is there little agreement, at present, about how spatial ability and visual imagery should be defined, but also, clear relationships between spatial ability and visual imagery, whatever they are, and mathematical learning have not been identified. (Author/MP)

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SPATIAL ABILITY, VISUAL IMAGERY, AND MATHEMATICAL LEARNING

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This essay briefly reviews four literatures. The first of these literatures is concerned with definitions of the terms 'spatial ability' and 'visual imagery'; the second is concerned with training studies in which attempts have been made to improve spatial ability or to encourage greater use of visual imagery in problem solving; the third pertains to sex differences in spatial performance; and the fourth is concerned with studies investigating relationships between spatial ability, visual imagery and mathematical performance. It is concluded that not only is there little agreement, at present, about how spatial ability and visual imagery should be defined, but also, clear relationships between spatial ability and visual imagery, whatever they are, and mathematical learning have not been identified.

The idea that mathematical learning is strongly influenced by spatial ability is not new (see, for example, Hamley, 1934; Barakat, 1951; Hills, 1957; Wrigley, 1957; Werdelin, 1958; and Smith, 1964). However, it was only during the 1970s that mathematics educators around the world began to explore systematically relationships which might exist between spatial ability, visual imagery and mathematical learning. This recent interest was undoubtedly stimulated by the dramatic upsurge in interest in spatial ability and imagery among psychologists, and also the publication of English translations of the seminal studies by N.A. Menchinskaya (1969) and V.A. Krutetskii (1976), of the Soviet Union. There were other influences, of course, too numerous to mention here. The result is that researchers from many countries have taken up the challenge. In addition to the studies by Menchinskaya and Krutetskii, contributions by P.M. van Hiele (1959) of the Netherlands, Alan Bishop (1973, 1979, 1980) of the United Kingdom, Michael Mitchelmore (1976, 1980a,1980b) of Jamaica, and Elizabeth Fennema (1974, 1979)

of the United States, have, in the view of the present writer, been especially important for mathematics educators.

Despite the increasing volume of studies attempting to establish links between spatial ability, visual imagery and mathematical learning, very few, if any, clear relationships have been identified. One of the main reasons for this is the absence of widely accepted definitions of 'spatial ability' and 'visual imagery'; this has resulted in serious doubts being expressed concerning the validity of tests purporting to measure these qualities (Egan, 1979). Even on the same spatial tasks different groups of subjects use qualitatively different processing strategies (Cooper and Podgorny, 1976; Tapley and Bryden, 1977). However, such difficulties have not prevented writers from using the notions of spatial ability and visual imagery in the formulation of hypotheses which, if true, have profound implications for mathematics education. McCallum, Smith and Eliot (1979), for example, have concluded that 'the single most important component of mathematical ability is a g/k factor which remains relatively stable in high school years and shows little relationship to language comprehension'. Julia Sherman, who has consistently included spatial variables in studies which have investigated influences on the mathematical performance of girls (see Fennema and Sherman, 1977, 1978; Sherman, 1979), has recommended that research be directed towards factors affecting the development of spatial skill, not only during early years, but even during adult years' (Sherman, 1979, pp.26-27).

This essay briefly reviews four important literatures which relate to the issue of how spatial ability, visual imagery and mathematical learning are linked. The first of these literatures is concerned with definitions of the terms 'spatial ability' and 'visual imagery'; the

second is concerned with training studies in which attempts have been made to improve spatial ability or to encourage greater use of visual imagery in problem solving; the third pertains to sex differences in spatial performance and the ways these differences might affect mathematical performance; and, finally, studies which have specifically examined relationships between spatial ability, visual imagery and mathematical performance will be surveyed.

DEFINITIONS OF SPATIAL ABILITY AND VISUAL IMAGERY

Spatial Ability

After reviewing the work of factor analysts over the last fifty years McGee (1979, p.909) concluded that there has been 'strong and consistent support for the existence of at least two distinct spatial abilities - visualization and orientation'. McGee indicated his approval for the following descriptions of Spatial Visualization (Vz) and Orientation (S) factors provided by Ekstrom, French, Harman and Dermen (1976):

> Vz: An ability to manipulate or transform the image of spatial patterns into other arrangements; requires either the mental restructuring of a figure into components for manipulation or the mental rotation of a spatial configuration in short term memory, and it requires performance of serial operations, perhaps involving an analytic strategy.
>
> S: An ability to perceive spatial patterns or to maintain orientation with respect to objects in space; requires that a figure be perceived as a whole.

These descriptions are, at best, vague and, at worst, self-contradictory. Consider, for example, the following two clauses in the description for S:

1. 'An ability to perceive spatial patterns', and
2. '... requires that a figure be perceived as a whole'.

In the expression 'ability to perceive spatial patterns' the use of the word 'pattern' suggests that a cognitive operation is required in order that the relationships between the elements forming the spatial pattern can be analyzed. However, this tends to contradict the second statement, which indicates that the figure has to be perceived as a whole. This contradiction is avoided if it is allowed that Orientation (S) involves both analysis of parts of a figure and perception of the figure as a whole, but if this is the case then the factor S would be very broad, and considerable overlap with Vz would appear to occur.

In a recent comprehensive review of the factor analytic literature Eliot and Hauptman (1981) drew attention to the work of Lohman (1979), who systematically re-analyzed data from major factorial studies (e.g. Thurstone, 1938; 1944; Guilford and Lacey, 1947; Guilford's structure-of-intellect data, 1967; Horn and Cattell, 1966), and reported that despite the seemingly inconsistent descriptions of spatial factors by different analysts, when a common theoretical and methodological perspective is assumed there is a remarkable consistency in factor structures since 1938. Lohman claimed that his re-analysis provided the strongest possible support for the existence of three spatial factors, which he called *spatial relations, spatial orientation* and *visualization*. According to Lohman *spatial relations* is defined by performance on tasks requiring subjects to rotate figures or objects mentally, and probably represents both the ability to solve the problems correctly as well as the amount of time a person takes to carry out the necessary mental rotations; *spatial orientation* involves the ability to imagine how a stimulus array would appear from a different perspective; the third factor, *visualization*, is defined by a variety of tests, including hidden figures and surface development tests, which share two features: all are usually administered under different and varied time limits, and

all are more complicated or difficult than those loading upon the *spatial relations* and *spatial orientation* factors. In addition to these three major factors Lohman found 'minor' spatial factors which are defined by performance on tasks requiring visual memory, speed in matching visual stimuli and speed in making left-right discriminations.

While Lohman's study represents a significant contribution to the factor analytic literature it is not yet clear whether it has made sense of over forty years of different interpretations of spatial factors among American psychologists. In the view of the present writer, Lohman's description of the factor he calls *visualization* is sufficiently vague to be almost meaningless; also, as Eliot and Hauptman (1981) have commented, Lohman's descriptions of his *spatial relations* and *spatial orientation* factors emphasizes the difficulty of measuring the latter factor because it is possible for someone to imagine how a stimulus array would appear from a different perspective by carrying out a mental rotation of the stimulus rather than by reorienting an imagined self.

European workers have, almost traditionally, rejected the American belief that a number of different spatial factors exist. In 1961 P.E. Vernon, then at the University of London, maintained that there was only one major underlying spatial ability and that the American 'plethora of spatial factors is more confusing than helpful' (Vernon, 1961, p.75). Three years later I. Macfarlane Smith, also of the University of London, reported data which, he claimed, indicated that a task is not spatial, in the psychological sense, unless it requires 'the perception of a configuration as a whole' (Smith, 1964, p.97). Smith criticized attempts to distinguish between spatial visualization and orientation, and claimed that not only had these attempts failed, but also they would continue to fail (p.95). In the light of this comment it is interesting to note that

the American writers Guay, McDaniel and Angelo (1978) have recently argued that gestalt processing, which is characterized by the formation and transformation of mental visual images as organized wholes, is the essence of true spatial ability, and that many well established pencil and paper tests of spatial visualization and orientation require only minimal amounts of gestalt processing but large amounts of analytic processing. If, indeed, Guay et al. are correct then the optimistic claim of McGee (1979) and Eliot and Hauptman (1981), that a careful review of the factor analytic literature strongly points to the existence of two or three spatial factors, becomes questionable; for factor analysts may have been continually basing their analyses on tests which, although called 'spatial', in fact measure, some unknown combinations of spatial and non-spatial traits. Egan (1979) appears to have supported this view when he stated that recent information-processing research on spatial tasks suggests a new interpretation of scores on traditional tests of spatial ability, an interpretation in which a response to an item 'is a composite reflecting the combined output of several operations' (p.10). Similarly, Carroll (1980), after examining 55 studies of individual differences and re-analysing 25 data sets through factor analysis, concluded that although evidence now exists for a relationship between elementary cognitive tasks and scores from conventional psychometric tests, the nature of this relationship remains unclear because the components of psychometric test scores have yet to be identified more specifically. Interestingly, it could be argued that these recent conclusions by Egan and Carroll were anticipated by Cattell (1971), Royce (1973), and Horn (1976) who proposed the existence of a second-order visualization factor which spans abilities measured by widely-used tests of spatial visualization, flexibility of closure, adaptive flexibility, speed of closure, and spatial scanning.

The lack of agreement among factor analysts on the nature of spatial ability led one European writer, Fairweather, to state in an article on 'sex differences in cognition' that 'it is clear that the term 'spatial' has been used to connote a motley collection of skills' (Fairweather, 1976, p.249). He went on to say that because so many different interpretations have been given to the term it is difficult to report any generalizations which can be safely made from the huge amount of research into spatial abilities which has been reported. Even McGee, who is much more optimistic about the situation, has admitted that there is 'still vast disagreement about just to how to classify standard tasks of spatial abilities', and that the existence of numerous studies which show positive correlations between tests of spatial visualization and orientation 'illustrates the need for further analytic research to clarify the specificity of the visualization and orientation factors' (McGee, 1979, p.893). It is the view of the present writer that the kind of research McGee is recommending is not likely to result in the clarification he desires.

Not only are there differences of opinion among factor analysts on questions concerning spatial abilities, but there are also other notions of spatial ability held by developmental psychologists and educationists (e.g. geography educators). While factor analysts have busied themselves in attempting to identify and refine different spatial factors and subfactors Piagetian researchers, at the Geneva school and elsewhere, have been investigating whether a person's ability to think about space develops in well-defined stages, from topological beginnings through projective and Euclidean stages, to a stage of maturity which is typically reached during early adolescence (Piaget and Inhelder, 1967).

The Genevan position on how spatial concepts develop in children is set out in three major works, *The Child's Conception of Geometry* (Piaget, Inhelder and Szeminska, 1960), *The Child's Conception of Space* (Piaget and Inhelder, 1967), and *Mental Imagery and the Child* (Piaget and Inhelder, 1971). Piaget and Inhelder emphasize the distinction between *perceptual* and *representational* thinking, and maintain that it is only when the latter type of thinking has been fully developed, so that mental images can be internally manipulated, that an individual can be said to be spatially mature. The distinction between *figurative* thinking, which is concerned with the perception of static patterns of objects and the formation of static visual images, and *operative* thinking, which is concerned with perception of pattern in the movement of objects and the ability to manipulate visual images, is also important for Piaget and Inhelder, who maintain that it is operative thinking which is vital to geometrical thinking.

Piaget and Inhelder's description of a spatially mature individual draws attention to the relationship between spatial and mathematical development. Children who have mastered tasks representative of the Euclidean stage are able to identify and remember spatial patterns among the objects by introducing their own frames of reference. In essence they can construct, mentally, three-dimensional co-ordinate systems, and are able to cope with imagined congruent and non-congruent transformations of figures.

Regrettably, there has been little communication between factor analysts and Piagetian researchers interested in spatial ability, but as Wattanawaha (1977, p.46) has noted, both groups believe that persons with well-developed spatial skills should be capable of imagining spatial arrangements of objects from different points of view and of

manipulating visual images. Thus, while the two groups have addressed themselves to answering different questions their work is, to some extent, complementary.

Piaget and Inhelder are not the only writers to have attempted to provide a theoretical basis for describing the development of spatial abilities. Werner (1964), has put forward his *orthogenetic principle* by which there is a progression from a state of relative globality and lack of differentiation to a stage of increasing differentiation, articulation, and hierarchic integration. Werner describes three levels of spatial development 'sensorimotor, perceptual and contemplative', and stresses that it is only at the contemplative level that extensive use can be made of visual imagery deriving from small and large scale space (See Hart and Moore (1973), and Bishop (1980) for further comments on Werner's work.). Other writers who have advanced theories which could explain the development of spatial concepts in children include Bruner (1964), with his well-known enactive, iconic and symbolic levels of representation, Mitchelmore (1980a), Siegel and White (1975), and Yi-Fu Tuan (1977).

Despite the many attempts to define different spatial abilities, and to describe how these abilities develop, there is certainly no evidence of unanimity in the literature on these matters. For example, data have been presented which not only cast doubt on Piaget and Inhelder's theory of how spatial concepts develop, but also on their description of what constitutes mature spatial ability. (See for example, Dodwell, 1963; Lovell, 1966; Bryant and Trabasso, 1971; Kapadia, 1974; Martin, 1976(a), 1976(b); Collins, 1976; Kidder, 1976; and Donaldson, 1979).

Nevertheless, Eliot and Hauptman (1981); after reviewing the work on spatial ability by differential and experimental psychologists, have concluded more positively, that it is reasonable to describe spatial ability 'as both the ability to organize the distributional aspects in the representation of information *and* as the ability to generate, retain and manipulate visual images'.

An interesting new approach to defining spatial ability, at present little known in U.S.A. and Europe, has been provided by Wattanawaha (1977), of Thailand. After an extensive review of the factor analytic and developmental psychology literature, Wattanawaha put forward her DIPT classification system for spatial tasks, and argued that this system could be used to provide an operational definition of spatial ability. She suggested that her system could provide a satisfactory basis for the selection of balanced sets of spatial tasks which would enable the quality of a person's spatial performances in different contexts to be predicted. Wattanawaha questioned the validity of many widely used spatial tests, especially those which contain a narrow range of tasks; such tests, she claimed, may not prove to be good predictors of performance on many important everyday spatial tasks.

Wattanawaha's DIPT system, which is intended for non-speeded tasks only, is based on the assumption that thinking about a spatial task can be summarized by attending to four independent characteristics: (i) the *D*imension of thinking required by the task, (ii) the amount of *I*nternalization required, (iii) the manner in which the answer has to be *P*resented, and (iv) the *T*ype of thinking required by the task, and in particular, whether the mental operations which have to be used for the task are given in its definition or whether they have to be worked out by the subject. (The underlined letters explain the origin of the term 'DIPT'.)

Associated with each of the four independent characteristics is a number of values, three for each of Dimension, Internalization and Presentation, and two for Type of thinking. Wattanawaha's definitions of value labels are given in Table 1.

Wattanawaha claimed that *any* spatial task intended for a well-defined group of people could be uniquely classifed by an ordering of the values of D, I, P and T. Thus, for example, a question with a classification of (2, 1, 2, 1) would be such that it required: (i) two-dimensional thinking, (ii) a visual image to be formed but not mentally manipulated, (iii) the answer to be a drawing or some other accurate description of a visual image; and (iv) the application of one or more mental operations which are implicitly, but not explicitly defined in the question.

It is surprising that Wattanawaha did not include *response latency* in her classification system; certainly, work by R.N. Shepard during the 1970s (see Shepard and Metzler, 1971; Shepard and Feng, 1972; Cooper and Shepard, 1975), and by Egan (1978, 1979) makes it clear that the time variable will play an increasingly important role in future research on spatial ability. Another criticism of Wattanawaha's approach is that she classified tasks rather than the responses to tasks by individuals. It is well known that different individuals often process the same spatial stimuli in different ways. Despite these criticisms, Wattanawaha's classification system has provided data of a kind not previously obtained (see the section on 'sex differences' later in this paper), and is worthy of the attention of all persons interested in spatial ability.

TABLE 1

Wattanawaha's DIPT Classification System for Spatial Tasks

Symbol	Name	Value Labels and Corresponding Definitions
D	Dimension (3 values) 1, 2, 3	1: A question requiring 1-dimensional thought. 2: A question requiring 2-dimensional thought, but not 3-dimensional thought. 3: A question requiring 3-dimensional thought.
I	Internalization (3 values) 0, 1, 2	0: The question can be done at the perceptual level. There is either no need for a visual image to be constructed, or the only visual image needed is a 'duplicate' of a given stimulus, or an image corresponding to a simple translation of all or part of the stimulus. 1: There is a need for a visual image to be constructed, but in order to do the question there is no need to transform the image in the mind. 2: There is not only a need for a visual image but in order to do the task this image must be operated upon (transformed) in the mind.
P	Presentation (3 values) 0, 1, 2	0: The expected answer form does not require a final visual image to be described, identified, or drawn on paper. 1: The answer is a picture which has to be identified from a number of different pictures which are presented in diagrammatic form, or are described by words or actions. The picture to be identified must correspond to the final visual image associated with the task. 2: The answer requires that the final visual image be represented by a drawing, or be described in words or by hand or other movements.
T	Thought Process (2 values) 0, 1	0: The task specifies the mental operation which needs to be carried out. 1: The task does not specify the mental operation but enough information is given for this to be determined.

Visual Imagery

Bourne, Dominowski and Loftus (1979) identified three general categories of visual imagery theories: picture analogy theories, propositional and symbolic representational theories, and surface representation theories. To these theories the present writer would add Paivio's (1971, 1973, 1977, 1978) dual coding theory (although Bourne et al. regard this as being a picture analogy theory). Because mathematics educators are increasingly displaying interest in the role of visual imagery in the processing of mathematical stimuli, it is important that they inquire into any implications for them which arise from definitional differences.

The picture analogy, or 'mental picture', view has been popular ever since the pioneering studies into imagery by Galton (1883). In his study, which was restricted to visual imagery, people were asked to visualize their breakfast tables and to describe the clarity of their images. Galton reported large individual differences, not only in the vividness of imagery but also in its controllability (i.e. the degree to which a subject could summon and direct it 'at will'), and subsequent research has, almost invariably, emphasized these aspects of visual imagery (see Richardson, 1977a; Evans, 1980).

According to the picture analogy theory, individuals store 'mental pictures' in their memories, and these pictures, which are replicas of previous sensory patterns, appear on the mind's screen in raw and uninterpreted forms (see Kosslyn and Pomerantz, 1977). This view has been the subject of strenuous criticism during the 1970s, with Pylyshyn (1973) being, perhaps, the strongest critic. Pylyshyn claims that information could not be stored in the form of unprocessed mental photographs, because there would be far too many and no possible means of organizing access to them. Thus, he

argues, recalled imagery is conceptual in nature and must derive from information stored in a more abstract form; therefore, the images formed are not themselves the process of imagery, but by-products of internal *abstract* representations. It should be noted, however, the Pylyshyn does not deny the existence of mental images; indeed he concedes that 'we cannot speak of consciousness without, at the same time, implicating the existence of images' (Pylyshyn, 1973, p.3).

Paivio (1971, 1977, 1978) has proposed a dual coding mechanism for memory, one code being visual and the other verbal in nature. Paivio, who has stated that in modern psychology no major researcher in imagery accepts the old 'mental picture' view as a working theory (Paivio, 1977, p.49), contends that two distinct symbolic systems are involved in perception, memory, language and thought. One system processes and generates nonverbal information (e.g. interpreting a photograph), and the other processes and generates verbal information, including all kinds of spoken and written language, and 'inner speech'. Although, in Paivio's theory, the two systems are independent of each other, in the sense that one can function without the other, they are also interconnected because nonverbal information can be mentally transferred into verbal information and vice versa. Thus, for example, verbal instructions to form images can be used as a device to assist learning. Kent and Hedger (1980) quote the following example of a geometry teacher who gave the following instructions to his class:

> Close your eyes, all of you. Now clear your screens. Bring a dot in from the left and fix it. Now expand a circle from that dot. What happens? ... Right, bring it back in till it disappears into the dot. Now fix a second dot and let two circles expand at the same rate from each. Describe the locus of the points where they meet.
>
> (Kent and Hedger, 1980, pp. 140-141)

The superiority of Paivio's dual coding scheme to the older picture analogy theory is made clear by this example, for although the children would not have had stored, in their memories, mental photographs of dots and circles, the task described would appear to have been perfectly reasonable.

The propositional or symbolic representation theory of imagery arose out of dissatisfaction with the dual coding theory. As Pylyshyn pointed out, a major weakness of the latter theory is that it does not explain how verbal information can be transferred, in the mind, into nonverbal information, and vice versa. This led to the proposal for a third kind of mental representation, one which would resemble neither pictures nor words, would not necessarily be accessible to subjective experience, but would explain how the transformations could occur. The propositional, or symbolic, representation theory put forward in the early 1970s (Simon, 1972; Anderson and Bower, 1973; Pylyshyn, 1973) was based on the idea that a person's knowledge could be represented in memory by sets of propositions, and the common format of these propositions would enable both verbal information and mental images to be generated. If a person wishes to retrieve information

from memory he searches for the proposition or propositions which represent the required information; this proposition is then transformed into verbal information or a visual image. In the case where the abstract representation system does not contain a proposition relevant to a required piece of informtion a person might try to deduce this proposition from those which are available. According to Pylyshyn the propositions should not be thought of as actual sentences in 'natural' language, and they are not accessible to subjective experience.

Somewhat ironically, Kosslyn and Pomerantz (1977) have criticised the propositional representation theory for the same reason that Pylyshyn criticised Paivio's dual coding theory. If Pylyshyn could ask how, according to the dual coding theory, verbal information and mental images could be transformed into each other, then Pylyshyn himself could be asked a similar question with respect to the propositional representation theory: how can information be transformed from an abstract representation into verbal information or a mental image? Kosslyn and Pomerantz also maintained that it was not clear that one needed to postulate the existence of an abstract form of representation to explain the transformation from verbal information to mental images, and vice versa. Kieras (1978) has suggested, for example, that verbal and nonverbal information can both be stored together in a common format in memory; further, since this common format is not abstract, every piece of information can retain its verbal or imaginal modality.

Kosslyn (1978) has proposed another theory, the surface representation theory, as an explanation for imagery representation. According to this theory, images are like displays produced on a cathode ray tube by a computer program plus data, and these images are temporary spatial

displays in active memory that are generated from more abstract representations in long-term memory. The 'mind's eye' looks at these internal displays and classifies them in terms of semantic categories (as would be involved, for example, in realizing that a particular spatial configuration corresponds to a dog's ear). While Kosslyn's 'more abstract representations in active memory' may sound like the abstract representation in Pylyshyn's propositional representation theory, in Kosslyn's proposal the same abstract format does not store and generate mental imagery and verbal information. Also, Kosslyn has not specified the nature of his abstract representations, whereas Pylyshyn's abstract representation contains propositions. In this sense, Kosslyn's theory is less detailed than Pylyshyn's, possibly because Kosslyn believes 'it is a virtue to begin with only the outline of a model, a proto model' (Kosslyn, 1978, p.294).

Despite the remarkable return to favour of imagery research in the 1970s, there is little agreement at the present time on the nature of imagery, and its origins within individuals. As Kosslyn has said, any detailed theory of imagery at this early stage 'is almost certain to be wrong' (Kosslyn, 1978, p.294). One of the main problems in imagery investigations is that of finding an appropriate research methodology. In the past, introspective evidence has often been used (see, for example, Hayes, 1973), but as Evans (1980) has shown, the use of introspections, even as corroborative evidence, is dangerous. Evans refers to findings by Wason and himself (Wason and Evans, 1975) as a basis for criticizing Kosslyn's acceptance of introspective reports as being useful, and then says of Kosslyn:

> It seems that he and like minds accept them if they achieve three criteria: clarity of expression, confident assertion by the subject, and consistency with the behaviour observed. In the experiments which I ran with Wason all three criteria were met on protocols that were nevertheless apparent rationalizations. Thus Kosslyn's notion of introspections as useful corroborative data is unsound (Evans, 1980, pp. 293-4).

Evans goes on to assert that there is a considerable range of evidence in social and cognitive psychology which suggests that introspective reports do not indicate awareness of underlying causal processes; rather, they consist of cause-effect type theories which subjects construct when attempting to explain their own behaviour (see, also, Nisbett and Wilson, 1977).

As in the case of spatial ability, developmental psychologists interested in describing the development of imagery in children have, for the most part, worked independently of other psychologists. In *Mental Imagery and the Child* Piaget and Inhelder (1971) distinguished *reproductive* images, in which known objects or events are evoked, from *anticipatory* images, in which objects or events not previously perceived are represented. They also noted that images can be immediate or deferred, and pointed out that it is only deferred images which are isolated from the sensory stimuli. Further, images can be *static*, *kinetic* or *transformation*.

Although developmental studies on imagery emphasise the role of action in assisting the development of imagery, the 'mind's eye' notion of visual imagery is always implicitly assumed in them. Interestingly,

the same is true of Kosslyn's surface representational theory. Indeed, in all the theories of imagery which have been discussed in the paper it is assumed that, in the case of visual imagery, the end product of whatever mental processes are involved is a mental picture. The controversy which surrounds the definition of imagery is not concerned with the end product itself, but rather with the processes by which the end product is achieved. This point is critically relevant, not only for those interested in describing relationships between visual imagery and spatial abilities, but also for anyone involved in designing programs aimed at fostering spatial abilities. If the ability to form and manipulate visual images is a necessary component of spatial maturity, and the review of the literature concerned with defining spatial ability would indicate this is the case, then the question of identifying the mechanisms by which visual images are evoked must be considered by all persons interested in constructing spatial training programs (see Eliot and Hauptman, 1981).

SPATIAL TRAINING PROGRAMS

Jean Piaget (see Piaget, 1964; Weinheimer, 1972) questioned the usefulness of training programs aimed at accelerating the rate of cognitive development in children, and indicated that if such programs were attempted then two criteria for determining their effectiveness should be adopted: (i) the retention criterion - were skills acquired by learners retained over a non-trivial period of time? and (ii), the transfer criterion - were learners able to use the skills they acquired in new problem solving situation?

There are numerous papers in psychological and educational journals and many other unpublished studies, in which results of spatial training

programs are presented and it is claimed that educationally significant improvements in children's spatial performances have been effected. Before the training literature is reviewed here it is worth emphasizing that very few spatial training studies have paid attention to the crucial matters of retention and transfer. Further, even if these two criteria are not applied, many of the results which have been reported are difficult to interpret because of inadequacies in the designs of the studies. Bruner (1973) maintained that we had not 'begun to scratch the surface of training in visualization', and Mitchelmore (1976), in a review of cross-cultural research, concluded that 'the greatest need is the development of practical geometrical and spatial teaching programs and for their experimental testing'. Apparently these sentiments are still true today, for Eliot and Hauptman (1981) conclude their review of the spatial training literature with the comment: 'At the present time ... we do not have results from long term training efforts which indicate whether training effects persist'.

In the review which follows, training studies have been classified into three groups: (i) studies in which a wide range of spatial tasks were included in the training program; (ii) studies in which the training program has consisted of a narrow range of tasks; and (iii) studies in which special spatial training programs were not provided, but the effects on spatial performance of experiences in certain subjects (for example, art and engineering drawing) were investigated.

Studies in which a wide range of spatial tasks were included in the training program.

The present writer has supervised a number of studies of this kind during the past five years, and since details of these can be found only in unpublished reports and theses it will be useful to describe, briefly, results which have been obtained.

After individually testing children in grades K-2 at an elementary school in Melbourne, Australia, on a number of Piagetian spatial tasks, Teggelove (1975) divided 48 of the children into sixteen matched triples and then randomly allocated each child in a triple to one of two treatment groups or to a control group. The control received no special training but children in the treatment groups were removed from class in small groups for eight thirty-minute training sessions over a period of eight weeks. One treatment group worked on a wide variety of pencil-and-paper spatial tasks which required students to form images of two-dimensional objects but did not require them to manipulate the images. The other treatment group became involved in an outdoor physical activity spatial program which, although it required two and three dimensional thought, rarely required mental manipulation of images. At the conclusion of the treatments the two treatment groups and the control group were given the same Piagetian tasks which had been used as a pretest and gain scores of students within each matched triple were compared. Although no statistically significant differences were found between the mean gain scores of the three groups, trends were in favour of the two treatment groups. Teggelove suggested that with the treatment groups he may have attempted to develop too many spatial skills in too brief a period.

Collins (1976) individually tested 200 grade 7 students and 60 grade 9 students on Piagetian 'cross-sections of solids' tasks (Piaget and Inhelder, 1967), and, on the basis of results on these tasks, and on pencil-and-paper tests, fifteen matched pairs of girls were formed and members of pairs randomly allocated to a treatment or a control group. The treatment group was given a six hours' program containing a wide variety of two and three dimensional spatial tasks, including tangrams and activities based on geometrical properties of plane and solid figures. After the program was completed the thirty girls were, once again, individually administered the cross-section tasks. While there was no statistically significant difference between the mean gain scores of the treatment and control groups, nine students from the treatment group improved more than their matched pairs from the control group, and only three from the treatment group did not do as well as their matched pairs from the control group.

Stephens (1979) gave eighteen grade 8 students (six boys, twelve girls) in a suburban high school in Melbourne the *Monash Space Visualization* pencil-and-paper test (Clements and Wattanawaha, 1978) and the Mitchelmore (1974) *Three Dimensional Drawing Test*. On the basis of results, six matched pairs were formed and members of pairs were randomly allocated to a treatment or control group. The treatment group received eight 50-minute training sessions during which special attention was given to identifying and modifying the thought processes of individuals. During the time the treatment group attended training sessions the control group attended Graphic Communication classes and received instruction on isometric drawing and the interpretation of Cartesian graphs. When the students were retested on the *Monash Space Visualization* and the *Three Dimensional Drawing* tests, the treatment

group did significantly better (two-tailed t-test , $p<.05$) than the control group on both tests. More qualitative analysis indicated that the thought processes used by children in the treatment group when attempting spatial tasks had been considerably modified.

The design of Wearne's (1979) study was almost identical to that of Stephens, the only difference being that Wearne used twelve low achievers on the *Monash Space Visualization Test* to form six matched pairs, and then randomly allocated members of pairs to form a treatment and a control group. The students were in grade 8 at a high school in Melbourne, and the treatment group received six hours of training in a wide range of spatial tasks, including multi-view problems, surface development questions, paper-folding and paper-cutting exercises, perspective tasks (including Piaget and Inhelder's (1967) 'three mountains task'), reflection tasks, the drawing of haptically perceived shapes, cross sections of solids, and the representation of three-dimensional solids by two-dimensional drawings. The treatment group's program was conducted by Wearne himself, during lunchtime sessions (during which time the control group children were at lunch). When both groups were given the *Monash Space Visualization Test*, once again, the treatment group did significantly better on it than the control group.

An interesting aspect of Wearne's study is that he made use of an 'analysis sheet', a pencil-and-paper instrument which he constructed in an attempt to determine thought processes used by persons answering the questions on the *Monash Space Visualization Test*. The 'analysis sheet' contained fifteen questions (e.g. 'Did you rotate the object in your mind?') to which 'Yes' or 'No' answers could be given, and, on

each of two occasions (pretest and posttest), subjects in the study were required to answer all of the fifteen questions on the analysis sheet for each of the thirty questions on the *Monash Space Visualization Test*. Wearne found that subjects who made greatest pre-post gains on the Monash test were those who, apparently as a result of the training program, became more 'analytical', in the sense of preferring verbal-logical to visual-pictorial procedures. Wearne suggested that this could have been the result of his own preference for verbal-logical explanations. A weakness of this aspect of the study is the absence of any technical valuation of the 'analysis sheet'; this would appear to be especially serious in the light of criticisms by Evans (1980) of evidence based on introspections.

While the random allocation of subjects to treatment and control groups is a strong feature of the designs of each of the above studies, there are a number of obvious weaknesses. No retention or transfer data were reported; the duration of training programs was from six to ten hours, only; the Hawthorne effect was not controlled; further, as White (1979) points out, in such experiments the psychological mechanism by which any improvements are effected is unclear - 'is it the mere *presence* of apparatus which improves the pupil's spatial ability, or the *activity* (kinaesthetic) involved, or the *task* set by the teacher, or indeed the *discussion* of the task with the teacher?' (see Bishop, 1973, p.48)

Rowe (in progress) has just completed an ambitious spatial training study which, because the training programs were incorporated into the normal grade 7 curriculum of a school, minimized the Hawthorne effect. Subjects were 200 grade 7 students at two high schools in neighbouring country towns in Victoria. The control group consisted of 73 pupils who constituted the entire grade 7 enrolment at one of the schools; there

were two treatment groups, each consisting of 63 pupils who were in grade 7 at the other school. Both training programs were of twenty hours' duration, one hour per week for twenty weeks, and consisted of many short spatial activities which Rowe had prepared. One program consisted of tasks which required two-dimensional thinking and the ability to form visual images of shapes, but not mental manipulations of images; the other program consisted of tasks requiring three-dimensional thinking and often the mental manipulation of visual images. Following Wattanawaha (1977), Rowe called her treatments the 'D=2 and I=1', and the 'D=3 and I=2' programs. The programs were supervised by the seven grade 7 mathematics teachers at the 'treatment' school; these teachers rotated around the seven grade 7 classes, so that each spent roughly equal amounts of time on both programs. The pupils were randomly assigned to treatment groups from matched pairs which had formed at the beginning of the year on the basis of results on a battery of pencil-and-paper spatial tests. Immediately after the training programs were completed a similar battery of tests was given, as posttests, to the treatment and control groups, and a year later tests were administered in order to provide retention data. Both treatment groups performed equally well on the posttests, and both showed significantly greater gains than the control group on all spatial tests; closer analysis indicated that the improvement of both groups was mainly on tasks which Wattanawaha would have classified, under her DIPT system, as having a 'Dimension' value of 2 and an 'Internalization' value of 1, that is to say, tasks which required two-dimensional thinking and no mental manipulation of images. Neither treatment groups showed a

significant improvement on 'D=3 and I=2' tasks, that is to say, tasks requiring three-dimensional thinking and mental manipulation of images, despite the fact that one of the programs had been especially designed to foster this type of thinking. The retention data are currently being analyzed, but it appears that the superiority of the treatment groups over the control group has been maintained.

Saunderson (1973), in a study conducted in Papua New Guinea, used results on a general ability test to obtain 46 matched pairs of teachers' college students, and then randomly allocated members of the pairs to a treatment or a control group. The treatment group was given a five-hour spatial program over a period of three weeks, while the control group continued with a 'normal' mathematics course. The training program contained many activities in which equipment such as soma cubes, plastic straws, tesselation tiles, mirrors, and games such as three-dimensional tic-tac-toe were used. Saunderson reported that when a variety of spatial tests, mostly involving the mental manipulation of 'two-dimensional' images, were given to the treatment and control groups after the program had concluded, the treatment group performed significantly better than the control group.

In a study conducted in which the subjects were English comprehensive school children aged about thirteen years, McCloskey (1979) found that although, in a pre-posttest comparison study, a treatment group consisting of eleven boys and six girls obtained a greater mean gain than a control group (fourteen boys, thirteen girls) on NFER *Spatial Tests* I and III, the difference between the mean gain scores was not statistically significant. The training program consisted of fourteen lessons given over a period of seventeen weeks and the control group studied colour, texture, poster design and portrait painting while the training program

was in operation. An important weakness in the design of this study was that subjects were not randomly allocated to the treatment and control groups; indeed, it appears that the groups remained intact for most of their school subjects, and if this was, in fact, the case then systematic but uncontrolled influences could have affected results. Also, it is possible that control group activities, such as training in poster design, assisted performances on posttests.

There are surprisingly few American studies in which training programs which consisted of a wide range of tasks have been used. Brinkman (1966) devised a programmed instruction course in elementary geometry, in which visual-spatial methods were encouraged, and gave this course to a group of 27 eighth grade students (14 males, 13 females) over a period of three weeks. During the course the students became involved in a range of activities (e.g. planning geometrical designs, cutting out nets of solids), and were expected to complete ten units, each of which contained 50 exercises. Brinkman included a control group in the design of his study, and during the three weeks this group took a programmed course in 'normal' mathematics. When the D.A.T. *Space Relations Test* was given to both groups at the inclusion of the programs, the mean of the 'visual-spatial' group was significantly higher than that of the control group. In his report of the study Brinkman made no reference to a retention test having been employed. Also, although Brinkman claimed that the two groups were matched on sex and scores on the *Space Relations Test*, he also stated that the two groups remained under the care of their usual mathematics teachers during the programs; this suggests that students were not randomly allocated to groups.

Wolfe (1970, cited by Eliot and Hauptman, 1981) provided a program similar to Brinkman's for children in grades 7, 8 and 9. The program, which was of four week's duration, had videotaped lessons interspersed with other student activities. Eliot and Hauptman comment that many of the lessons appeared to parallel tasks used in the spatial tests which the students took. Wolfe found significant gains on problem related tests, but little gain in scores on 'transfer' tests.

It is the view of the present writer that in the future the most effective spatial training programs for schoolchildren will be those which offer a wide range of carefully sequenced tasks. Ideally, such programs will be incorporated into the normal curriculum, and will be based on findings of educational and psychological research concerned with imagery and spatial ability. With this in mind, the writer would maintain that studies of the kind described above have been the most useful of all spatial training studies carried out thus far. The wide range of tasks employed, and the fact that, in most cases, the tests used did not parallel tasks in the programs, meant that a 'transfer' aspect was indirectly incorporated in the studies. Unfortunately, most of the studies did not include retention tests, the study by Rowe being a notable exception. Rowe's use of Wattanawaha's DIPT system in designing balanced sets of tasks for her training programs is also especially noteworthy.

Studies in Which Training Programs Have Provided a Narrow Range of Tasks

There have been a large number of studies of this variety, many of which have been published. However, most of the studies employed neither retention nor transfer tests and there is some doubt about the educational value of many of the training programs, especially since, in

many cases, the tasks were very similar to those on the tests used to assess the effectiveness of the programs.

The best studies in this category are those reported by Cox (1977a, 1977b, 1977c, 1978) in which five year-old English children were given training in selecting a picture representing the view of an object as seen by an interviewer in a different position. In each experiment a group of children who did not perform the task correctly was identified, and from this group matched treatment and control groups were formed. Each member of the treatment group was given twenty 15-minute individual training sessions, in which small, structured steps of increasing complexity were provided. It was shown that either verbal or visual feedback, informing the child of correct or incorrect responses, was effective, with treatment groups significantly outperforming control groups on posttests. Retention tests given seven months after the training sessions were completed showed that the treatment group retained its superiority. Cox also tested if transfer to other tasks had occurred and found that although some transfer to other perspective tasks was evident, transfer to tasks such as *Progressive Matrices,* and water-level and cross-section tasks was very limited or non-existent.

Other studies in which children have been trained on perspective tasks include those by Arond (1966), Eliot (1966), Miller, Boismier and Hooks (1969), and Miller and Miller (1977). In all cases the subjects for these studies were aged between five and eight years, and although treatment groups showed significantly greater improvements than control groups in all but one of the studies (Eliot's being the exception) in each case the training program appeared to be carefully designed so that children's performances on the criterion tests would improve.

While it would appear to be legitimate to maximize the effeciency of training programs by helping subjects develop appropriate strategies for coping with criterion tasks, one wonders, in such cases, about the educational value of such programs; surely, one should be surprised if, under such conditions, treatment groups did not improve on the criterion tests. But will the subjects be able to transfer their learning to *other* spatial tasks? In the studies which have been mentioned in this section, 'transfer' tests were rarely used. The question of how particular or how general a spatial training should be is one which deserves more attention; and its answer will only become clear when a substantial body of data from 'transfer' studies becomes available. Connor, Schackman and Serbin (1978) have provided an example of the kind of research which could be useful; they claimed that a brief training period on embedded figures tasks positively influenced children's performances on a surface development test. In another study, involving 112 children aged 7 to 13 years, Guay (1977) tested the hypothesis that the skills involved in surface development tasks are among the pre-requisite skills for successful completion of multiple perspective tasks. He found that children tended to do better on multiple perspective tasks if they had already done a surface development test, but the reverse was not true.

Space does not permit the detailing, here, of all known studies in which attempts have been made to improve performance on particular spatial tasks, so brief summaries of a few representative studies will be given. Smith and Schroeder (1979) gave ten 30-minute classroom lessons on solving tangram (tesselation) puzzles to three grade 4 classes, and found that students who received the instruction significantly outperformed students

in a control group on a criterion test consisting of tangram puzzles; Rovert (1974) and Vandenberg (1975) designed treatments which improved the performances of children in grades 3 and 6, respectively, on object rotation tasks; and Salomon (1974) found that both a 15-minute movie film depicting the folding and unfolding of the nets of five different geometric solids, and a set of slides depicting the same objects with their nets were effective in improving the performances of ninth grade students on a surface development tasks, with the movie having the greater effect. While the results from this sample of studies are encouraging, there remains a concern about the educational value of some programs which are designed for the specific purpose of helping students improve their performances on particular spatial tasks.

It is difficult not to be impressed, however, with the need for intervention programs aimed at assisting people in formal educational institutions in developing countries to overcome difficulties which are often experienced with pictorial and diagrammatic representations (Bishop, 1979; Clements and Lean, 1981; Deregowski, 1971; Hudson, 1960; Mitchelmore, 1980b; Munday-Castle, 1966; Nicholson and Seddon, 1977). There have been a number of reports of successful attempts to train people in the recognition of pictorial depth perception (Dawson, 1967; Forge, 1970; Duncan et al. 1973; Davies, 1973; Deregowski, 1974; Leach, 1975; Nicholson, Seddon and Worsnop, 1977), although Mitchelmore (1974) attempted, without much success, to train 414 prospective elementary school teachers in Jamaica to draw two-dimensional representations of three-dimensional objects. In Mitchelmore's study the students (194 men, 220 women, mean age 22.8 years) were randomly

allocated to a treatment or a control group; the treatment group took a unit in which they designed, constructed, and sketched models of solids while the control group studied a unit on statistics. Both units were individualized and continued for 12-16 periods over 3-4 weeks. Mitchelmore subsequently found no statistically significant differences between the treatment and control groups on his *Three Dimensional Drawing Test* (Mitchelmore, 1976). He suggested that the experimental group learned mostly the names of shapes with which they were already familiar, and learned little new about their spatial properties. In contrast to this result, Leach (1978) successfully trained grade 3 children in African schools in pictorial depth perception. The children learned that an object appears to diminish in size as it recedes from the observer; a further object may be overlapped by a nearer one; the nearer to the observer an object is, the lower the horizontal plane on which it stands; and parallel lines appear to converge as they recede from the observer. There is clearly, great scope for the creative design of programs of this kind, and the use of such programs should not be confined to developing countries.

The Effects of Various Curricula on Spatial Ability

Since the early 1940s there has always been some interest in whether material covered in certain subjects in school or tertiary curricula assist the development of spatial ability. Churchill, Curtis, Coombs and Hassell (1942) found only a slight but nevertheless statistically significant improvement in performance on the *Surface Development Test* by a group of engineering students who had spent nine weeks on a draftsmanship course. Faubian, Cleveland and Hassell (1942) reported,

however, that Air Force recruits who had been given training in drafting and blueprint reading did not do better than a matched control group on the *Surface Development Test*. Ranucci (1952) and Brown (1954) found that courses in high school geometry did not assist performance on spatial tests, and Mendicino (1958) reported that 150 grade 10 students who took a year-long vocational mechanical drawing course did not show a statistically greater improvement on the D.A.T. *Space Relations Test* than a matched control group. Thus, it would seem that there is no guarantee that courses which apparently make substantial use of spatial ability will assist the development of the ability.

There have been some more encouraging results, however. Blade and Watson (1955) traced changes in performance on the CEEB *Spatial Relations Test* of five groups of students over a period of four years, and found that the three groups which consisted of engineering students only showed significant gains after one year. Interestingly, though, these gains apparently reached a plateau and further improvement was not reported for the remaining years. Bishop (1973) compared performances on NFER *Spatial Test* I of four groups of seventh grade students in England, and found that students who, in previous years, had had extensive experience with manipulative aids in mathematics classes did best. This result led Bishop to use manipulative materials in a training program with secondary school pupils and, according to Bishop (1980), this resulted in 'some quite high gains in their spatial task scores'. Herman and Siegal (1978) reported a study in which children in grades K-5 walked through a 'model village' the size of a classroom and then attempted to arrange the components of a much smaller model of the village; after repeated walks and trials the youngest children improved to the level of the older children.

This sample of studies suggests that spatial ability and visual imagery are often required in topics in standard school or tertiary subjects, and this is especially the case when learners are actively involved using manipulative materials. This raises the possibility of teachers consciously choosing activities which might assist the development of spatial ability and the greater use of visual imagery. In a remarkable paper, Yokochi (1980) maintains that in Japan kindergarten children aged 4 and 5 are given elaborate spatial training courses which enable them, for example, to draw pictures in perspective, to understand the notions of parallel lines and direction, and to draw the nets of three-dimensional solids (e.g. a triangular prism). These examples suggest that so-called 'established findings' of developmental psychology may have resulted in Western educators underestimating the spatial potential of children.

Attempts to Improve Imagery Skills

Unfortunately, attempts to facilitate the use of imagery have usually been confined to fairly narrow contexts. Thus, Atkinson (1975) used imagery mediation in mnemonic techniques to assist learning of foreign language vocabulary; Ackerman (1975) developed an imagery technique for teaching sight-sound associations in reading; Peterson and Hancock (1973) used a figural mode (which, presumably, induced imagery) as well as verbal and symbolic modes to teach network training in freshmen mathematics, and found all three modes led to high levels of retention; Gropper (1966) found that a visual mode of presenting hydrostatics enabled learners to transfer acquired skills to rule application problems that were verbally presented, and that a verbal mode of presentation was less successful in promoting transfer to test items that were visually

presented; Salomon's (1972, 1974) experiments, in which viewing of a movie film greatly assisted students' performance on the *Surface Development Test*, suggested that audio-visual presentations can facilitate desirable use of imagery; and Richardson (1967) demonstrated that mental rehearsals and mental practice of athletic skills can lead to improved performance.

However, as Wicker (1978) comments, 'imagery is not likely to be a magic elixir that lubricates everything in mental life'. Even in terms of memory there is evidence that imagery mediation can slow retrieval during recall (Corbett, 1977) and that it does not aid long-term retention (Hasher, Riebman, and Wren, 1976). Some (e.g. Hollenberg, 1970) have claimed that it can have detrimental effects on abstract conceptualization (but see Anderson and McGaw, 1973).

Educators are beginning to relaize the potential of training to improve imagery (Ausburn and Ausburn, 1978). The Visual Education Curriculum Project of the Australian Curriculum Development Centre has, for example, recently produced a handbook for teachers entitled *Just Imagine ... Learning Through Mental Imagery: A Guide for Teachers* (Visual Education Curriculum Project, 1979) which is specifically aimed at assisting teachers to develop teaching strategies which will encourage their students to make greater use of imagery. But although psychologists have talked about training to improve imagery ability ever since Galton (1883), there is little evidence of progress at either the theoretical or pedagogical level. The theory advanced by Gagné and White (1978), that the effectiveness of imagery-evoking stimuli, such as pictures, diagrams and demonstrations involving real objects, is likely to be improved when careful attention is paid to the descriptive words that link them to other memory structures, would appear to be especially worthy of investigation.

We close this section with a question. Is it the case that two essential prerequisites to the construction of effective spatial training programs are (i) an understanding of how to facilitate the use of imagery, and (ii) a knowledge of those situations in which the use of imagery is likely to be advantageous?

SEX-RELATED DIFFERENCES IN SPATIAL PERFORMANCE

There is overwhelming evidence that in most cultures sex-related differences in favour of males appear from adolescence onwards on many tasks which require use of so-called 'spatial' skills (Maccoby and Jacklin, 1974; McGuiness and Pribham, 1978; Harris, 1978; Fennema, 1979; McGee, 1979). Questions which are raised by this phenomenon include:

1. What cognitive characteristics are there common to skills encompassed by the term 'spatial' skills?
2. Why, from adolescence, do males perform better than females on tests of spatial abilities?
3. Are there certain classes of spatial tasks which adolescent and adult females do as well as, or better than, males, and if so, do these tasks have identifiable common characteristics which distinguish them from spatial tasks on which males perform better?
4. Can special spatial training programs for women and girls be devised, so that sex-related differences in spatial performance would be reduced, eliminated, or even reversed?

Harris (1978) and McGee (1979) have comprehensively reviewed the literature relating to the second of these questions, discussing neurological, hormonal, environmental and genetic theories which have been advanced to explain superior male performance. Although neither review provides an

answer to the question no further discussion of it is provided in this essay. While the literature pertaining to the first question has already been reviewed, we shall return to it at various points of our discussion of the third and fourth questions.

On Which Spatial Tasks Do Sex-Related Differences Appear?

Harris (1978) discusses many tasks on which sex-related differences have appeared. According to Harris, sex-related differences on logical conservation, water-level tests, music composition, chess, geometrical and other mathematical tasks, 'sense of direction' problems, problems involving 'kinetic mental manipulation', embedded figures, and many other tasks, might all have their origin in sex-related differences in spatial ability. Taylor (1960) and Smith (1964) maintained, however, that girls are superior on problems requiring attention to detail whereas boys are better if the representation of proportions is required. Many writers (e.g. Vandenberg, 1975) have claimed the sex-related differences are especially likely to occur if mental manipulations of visual images are required; recently, however, Kail, Carter and Pellegrino (1979) claimed that adult males and females performed mental rotation tasks in the same ways, with females rotating the images at a slower rate than males. Williams and Brekke (1979), after reporting data for which no sex-related differences appeared, suggested that in the past differences may have been over-emphasized. By contrast, Buffery and Grey (1972) maintained that only about one female in twenty performs as well as the average male on putative tests requiring mental manipulations of images. A recent study by Richmond (1980), on the other hand, indicated that sex-related

differences may be greater on tasks requiring the use of operations than on tasks which depend on spatial ability as an intrinsic aspect of representation.

Clearly, the literature is not easy to interpret, and it was for this reason that Wattanawaha (1977) embarked on a large investigation into the types of pencil-and-paper spatial tasks on which sex-related differences are likely to appear. She used a stratified sampling procedure in selecting 2346 subjects (1201 males, 1145 females) in grades 7-9 in 73 Victorian schools, and then gave this sample a wide range of pencil-and-paper tasks, each of which, according to a Rasch analysis (Wright and Mead, 1977), measured the same trait (which, on the basis of correlations with other tests, was presumed to be a 'spatial' trait). The male-female ratio for grades 7-9 in each of the 73 schools was reflected in the male-female ratio in the sample for that school, and the urban-rural population for Victoria was also reflected in the sample. Respective proportions of students attending government high schools, government technical schools, non-government technical schools and non-government (non-Catholic) schools were also reflected in the sample. Students in the sample were aged from 11 to 15 years.

Students in grades 7 and 8 were given *Monash Space Test (1)* which contained 24 questions, and students in grade 9 were given *Monash Space Test (2)*, which also contained 24 questions, eleven of which were identical to questions on *Monash Space Test (1)*. Table 2 shows comparisons of male and female performances on questions on the tests, for each of grades 7, 8 and 9. Note that for each comparison a 2-tailed z-test for the difference between proportions (Ferguson, 1976, pp.173-5) was used to test the null hypothesis that no difference between the proportions existed ($\alpha = .05$).

TABLE 2

Comparisons of Performances of Male and Female Students in Grades 7-9 on a Range of Spatial Tasks

Grade	Test Used	Number of Questions on which ... Males did sig. better	Females did sig. better	Neither sex did sig. better
7	*MST 1*	7	0	17
8	*MST 1*	7	0	17
9	*MST 2*	11	0	13

The table shows that although at each grade level the girls did not perform significantly better than the boys at the same level on any question, for 47 of the 72 comparisons there were no statistically significant differences in the performances of the sexes. For 25 of the comparisons, however, the boys performed significantly better than the girls.

Wattanawaha used her DIPT classification system (see pp. 10-12 of this essay) as the basis for investigating characteristics of questions on which sex-related differences did, or did not, appear. She found that 24 of 28 comparisons on questions which had an I-value (Internalization) of 0 or 1 did not produce statistically significant sex-related differences. More particularly, questions for which the D-value (Dimension of thinking) was 2 *and* the I-value was 0 or 1 were likely to be equally difficult for males and females. Figure 1 shows one such question. This question, like many 'D = 2 *and* I = 0 or 1' questions which appear on many so-called 'spatial' tests, requires attention to detail. At the grade 7 level 64% and 69% of boys and girls, respectively, answered it correctly, while at the grade 8 level the corresponding percentages were 69% and 75%. (In both cases the proportions were not statistically significantly different.)

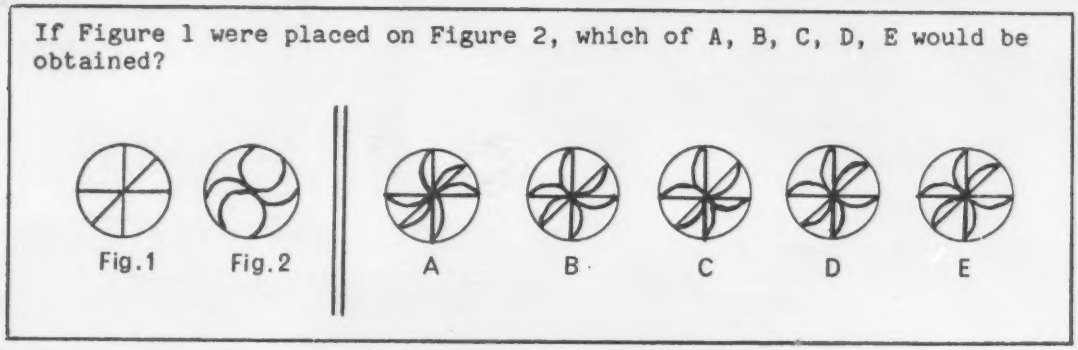

Fig. 1: A Question with DIPT Classification (2,1,1,0)

Wattanawaha also found that 17 of the 25 questions on which boys performed significantly better than girls were such that D = 3 *and* I = 2, that is to say, they required both three-dimensional thinking and kinetic manipulation of imagery. Figure 2 shows a question of this variety from *Monash Spatial Test (1)*. Boys in grades 7 and 8 did much better than girls in the same grades on this question.

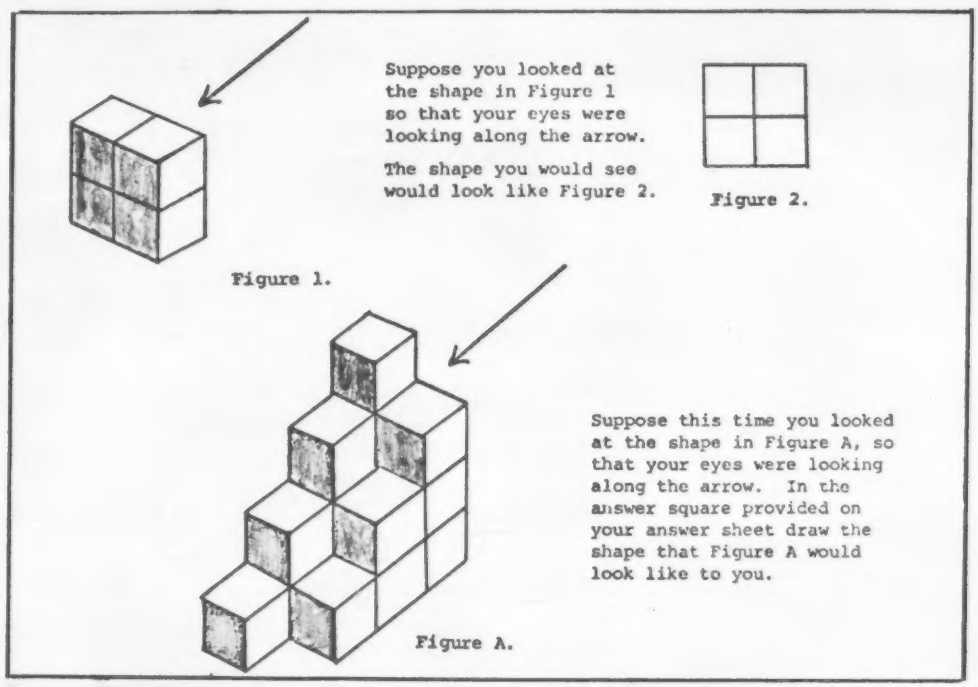
Suppose you looked at the shape in Figure 1 so that your eyes were looking along the arrow.
The shape you would see would look like Figure 2.
Figure 2.
Figure 1.
Suppose this time you looked at the shape in Figure A, so that your eyes were looking along the arrow. In the answer square provided on your answer sheet draw the shape that Figure A would look like to you.
Figure A.

Fig. 2: A 'D = 3 *and* I = 2' Task Favouring Males

Data in Wattanawaha's study indicated that adolescent males in Victoria were likely to perform spatial tasks requiring three-dimensional thinking or mental transformation of images better than females of the same age. It is emphasized, however, that for the majority of questions on *Monash Spatial Tests (1)* and *(2)*, significant sex-related differences in performance did not appear. Even questions with a D-value of 3 *and* an I-value of 2 were often answered as well by females as males. Fifteen of the 28 sex-related comparisons on 'D = 3 *and* I = 2' questions favoured males and thirteen did not.

Consideration of 'D = 3 *and* I = 2' questions for which sex differences did or did not appear in the Wattanawaha study suggested that the *context* of the question had an important influence on whether males performed better than females. One of the questions on *Monash Spatial Test (1)* involved thinking about paper-folding, which is not obviously a sex-biased task, and it is interesting that it was done equally well by boys and girls in grades 7 and 8. By contrast, however, boys in grades 7 and 8 did much better on the 'D = 3 and I = 2' question shown in Figure 2. There were a number of questions on the two Monash tests which required thinking about piles of blocks and, almost invariably, boys did better than girls on these. Such tasks would appear to be biased in favour of males.

This issue of test bias becomes critically relevant when the type of questions which appear on many widely used spatial tests is considered. The D.A.T. *Space Relations Test*, for example, contains only items which require the matching of a given two-dimensional net with a set of illustrated three-dimensional solids. In terms of the DIPT system, each item is such that D = 3 *and* I = 2. Given the context of the questions and their DIPT classification, one would expect males to perform better than females on the test. Published test norms demonstrate that this is indeed the case.

Tittle (1973, p. 118) has said: 'The content of an educational test should not reflect a stereotyped view of cultural groups, but should reflect equal opportunity for, and non-discriminatory views of, all cultural subgroups'. Is the D.A.T. *Space Relations Test* discriminatory, or is it, like many other spatial tests, simply an instrument which shows that males are superior in the ability which the test represents? Further, it must be remembered that the Monash tests contained a wide range of spatial questions, set in many different contexts, yet females at any of the three grade levels did not significantly outperform males at the same grade level on a single question. This indicates, in our view, that the spatial abilities of adolescent females in Victoria were generally not as well developed as those of adolescent males of the same age. However, the truth or otherwise of this statement depends on the meaning of the term 'spatial ability', and, as was shown earlier, an acceptable definition of this term has proved to be very elusive.

Spatial Training Programs for Females

An obvious issue arising from the above discussion is whether spatial training programs for women and girls can be devised so that sex-related differences in spatial performance are eliminated. Although there have been a number of studies which reported differential responses by males and females to spatial training programs (see, for example, Connor et al. (1977), Goldstein and Chance (1965) and Vandenberg (1975). McGee (1979, pp. 900-1), in briefly reviewing the literature on this matter, concluded that while 'there is some evidence that improvement in perceptual judgments occurs as a function of controlled practice and training ... ordinary school curriculum offerings are not always effective in developing spatial perception to the asymptote of an individual's ability'. According to McGee empirical research does not strongly support the hypothesis that females gain more from spatial training programs than males. Sherman (1979,

pp. 26-27), in the following passage, leaves no doubt on her feelings on the matter, however:

> Research needs to be directed towards factors affecting the development of spatial skill not only during early years, but even during the adult years. We do not assume that an illiterate adult should be written off as unteachable, nor should we assume that adults cannot improve their spatial skill. Methods of achieving this (and any improvement of spatial skill) need to be devised, and their feasibility and advisability evaluated. We know very little about maximally developing spatial skill. What contribution do crafts, shop classes, drafting courses, sports and/or other activities make to the development of spatial skill? Contributions of biological factors, though small, still remain to be clarified. The educational system must address itself to the issue of maximally developing female intellectual competence. Among other factors, this will need to include better development of spatial visualization skill. For a swimmer with a weak kick we provide a kick board and opportunities to develop the legs. We do not further exercise the arms.

Fennema (1979), who has written a number of major papers with Sherman (see, for example, Fennema and Sherman, 1977, 1978) is more guarded on the issue, however, maintaining that current studies will soon provide more information on whether intervention programs can be developed which will improve spatial skills. Wattanawaha's study would suggest that those who are concerned that women and girls, especially, should receive spatial training programs would do well to insist that any programs which are devised include a wide variety of tasks for which three-dimensional thinking and the mental manipulation of visual images are required.

RELATIONSHIPS BETWEEN SPATIAL ABILITY, VISUAL IMAGERY AND MATHEMATICAL LEARNING

In view of the fact that, at present, universally accepted definitions of spatial ability, visual imagery, or even mathematics, do not exist, it is hardly surprising that there is very little direct discussion in the existing literature on *why* spatial ability, mental imagery and mathematical performance might be expected to be related (note, though, Skemp, 1971; and Hayes, 1973). According to Fennema (1979), however, the relationship between mathematics and

spatial visualization is logically evident. She points out that, in mathematical terms, spatial visualization requires rotation, reflection or translation of rigid figures, which are important geometrical ideas; she also claims that 'many mathematicians believe that all mathematical thought involves geometrical ideas', and comments that 'therefore, if spatial visualization items are geometrical in character and if mathematical thought involves geometrical ideas, spatial visualization and mathematics are inseparably intertwined' (pp. 392-3).

It is not clear whether Fennema's assertion is correct. She, herself, has admitted that results from empirical studies which have explored the relationship between spatial visualization and mathematics are not consistent. Krutetskii, the Soviet psychologist and mathematics educator who spent many years clinically investigating the mathematical abilities of schoolchildren, concluded that spatial ability is *not* a necessary component of the structure of mathematical giftedness, though its presence alters the mathematical cast of mind (Krutetskii, 1976, p. 351).

Sherman (1980, p. 481) has hypothesized that more females than males develop their verbal facility and rely on this skill when a spatial approach might be more effective, which is consistent with Krutetskii's claim that some children develop strong preferences for using verbal-logical, as distinct from a visual-pictorial mode of thinking when attempting mathematical problems, whereas the reverse is true for other children. In the same vein, Tobias (1978) has provided an introspective account of how verbal facility interfered with mathematical learning. In this light, and bearing in mind that Hadamard (1954), Menchinskaya (1969), Poincare (1963), Richardson (1977b) and Walter (1963) have all contended that individuals can be classified into three groups with respect to a visual-verbal dimension, namely 'visualizers', 'verbalizers', and the 'mixers', it is interesting that both Moses (1977) and Lean and Clements

(in press) have not only assumed the existence of a visual-verbal processing continuum, but have also attempted to show how someone's position on this continuum can be located and related to performance on spatial and mathematical tests.

In their study Lean and Clements gave 116 engineering students in Papua New Guinea a battery of mathematical and spatial tests and, in addition, determined their preferred modes of processing mathematical information by means of an instrument developed by Suwarsono (in progress), an Indonesian doctoral candidate at Monash University. It is not possible to give further details of the study here, but two of the conclusions reached are of interest: (i) spatial ability and knowledge of spatial conventions had only a small influence on mathematical performance, and (ii) there was a tendency for students who preferred to process mathematical information by verbal logical means to outperform more visual students on *both* mathematical and spatial tests. With respect to the second point, it is worth noting that the student who gained the highest scores on the spatial ability tests showed no inclination at all to use visual imagery when solving mathematical problems.

The results of the Lean and Clements study would appear to be in conflict with other studies (e.g. Moses, 1977; Webb, 1979) which suggest it is desirable to use visual processes when attempting mathematical problems. However, they are perfectly consistent with Hollenberg's (1970) position that imagery mediation can have detrimental effects on abstract conceptualization. ATI findings of Salomon (1971) and Threadgill-Sowder and Juilfs (1980), which suggest that high-aptitude students experience interference when given treatments which provide them with mediators they can provide on their own, whereas low-aptitude students benefit when mediators they are lacking are provided overtly, would also appear to be relevant in this context.

CONCLUSIONS

The following conclusions would appear to be warranted from the reviews of the literatures which have been given.

1. At present there are no widely accepted definitions of the terms 'spatial ability' and 'imagery'.
2. Although attempts to define spatial ability usually make use of the notion of imagery, the literatures concerning the two constructs have developed almost independently of each other.
3. While training programs aimed at improving performance on a particular type of spatial task are likely to be successful, the educational value of such programs is questionable.
4. Although psychologists have talked about training to improve visual imagery ever since Galton (1883), there is little evidence of progress at either the theoretical or pedagogical level.
5. Adolescent males outperform adolescent females on many spatial tasks which require three-dimensional thinking *and* mental manipulation of images. Tasks which demand only two-dimensional thinking and do not require mental manipulation of images are not likely to produce significant sex-related differences in performance.
6. The importance or otherwise of spatial ability in the processing of mathematical information is not clear.
7. Empirical research does not strongly support the hypothesis that females gain more from spatial training programs than males.
8. In some circumstances imagery mediation can have detrimental effects on abstract conceptualization in mathematics.

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